**AN-700**
Application Note

# SIMULATE MECL SYSTEM INTERCONNECTIONS WITH A COMPUTER PROGRAM

*Prepared by:*
**Jerry Prioste**
**Tom Balph**
**Bill Blood**
Applications Engineering

Circuit interconnections are an important part of system design when using high speed logic circuits. The design of interconnecting paths affects both system speed and system accuracy. This application note describes the use of a computer program to simulate interconnections for high speed digital systems.

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# SIMULATE MECL SYSTEM INTERCONNECTIONS WITH A COMPUTER PROGRAM

Every digital logic family has design rules to define proper use of the parts in a system. With high speed logic, circuit interconnections become an important part of the design rules. Since the propagation time of a signal line may be a significant portion of system cycle time, system performance is dependent on the interconnection paths. System accuracy is also affected because signal distortion due to reflections can cause erroneous circuit operation. A technique for determining signal distortion during the design stage of system development is therefore a powerful aid to good system operation.

The physics of a signal on a line have been well defined. The termination techniques commonly used in RF signal transmission may also be used to improve digital system performance and insure waveform integrity throughout a system. This application note develops a technique for simulating system interconnections with a computer program. An included plot routine allows the user to observe a computer simulation of the waveform at the receiving end of a signal line. The program also enables the user to add parallel or series termination resistors to the signal lines to determine the waveform improvement due to using termination techniques. With a program of this type it is possible to computer simulate the performance of signal lines in a high speed digital system prior to system construction.

## TRANSMISSION LINE CONSIDERATIONS

The waveform of a signal at the end of a transmission line can be determined by calculating the reflection amplitudes. Figure 1 shows a test to measure reflections. A pulse generator terminated in a 50 ohm load drives a 75 inch, 50 ohm, unterminated line. The line is driven from a 10 ohm source impedance to simulate a MECL circuit output. The pulse generator amplitude at point A is set to give a 1 volt pulse amplitude at points B and C after the reflections have settled.

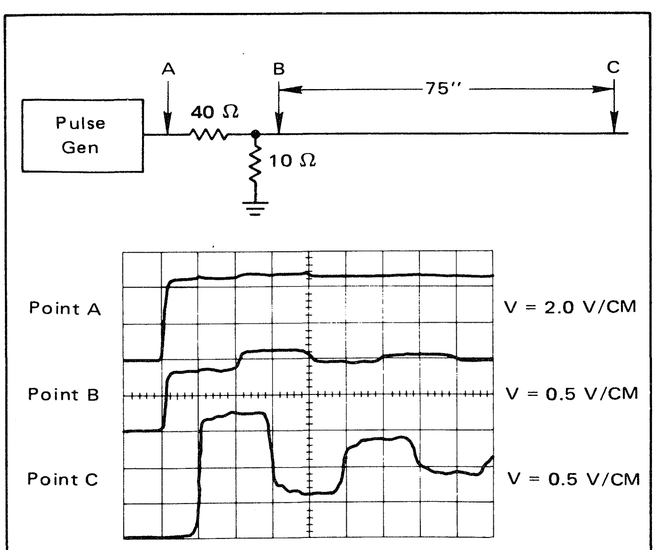



FIGURE 1 — Signal Line Reflection Test

The initial signal amplitude at point B is determined by the driving source impedance and the line impedance. The pulse generator used has a high impedance current source output. The steady state, one-volt output across 10 ohms requires a 100 mA generator output. The initial load at point B is the parallel value of the 10 ohm resistor and the 50 ohm transmission line.

$$R_{Li} = \frac{(50)\,(10)}{50 + 10} = 8.33 \text{ ohms}$$

A generator current of 100 mA and 8.33 ohms gives an initial signal amplitude of 833 mV. This 833 mV signal travels down the line and is seen at point C one propagation time later (10 ns). If not properly terminated the signal will reflect back down the line toward point B. The amplitude of the reflection coefficient at the load end of the line is calculated with the following equation. (For an open line $R_L$ is very large and the reflection coefficient $\rho_L$ is approximately +1.0.)

$$\rho_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1 - \frac{Z_o}{R_L}}{1 + \frac{Z_o}{R_L}} \approx +1.0.$$

This reflection is seen at point B two line-propagation-delay-times after the start of the initial signal. The reflected signal is then reflected toward point C as determined by the sending and reflection coefficient $\rho_S$. For the circuit in Figure 1 with an output impedance $R_o$ of 10 ohms, $\rho_S = -0.67$.

$$\rho_S = \frac{R_o - Z_o}{R_o + Z_o} = \frac{10 - 50}{10 + 50} = -0.67.$$

Figure 2 shows calculations to determine signal amplitudes at points B and C as a function of time. Comparing the calculated amplitudes in Figure 2 with the measured signals in Figure 1 shows a good correlation between measured and calculated signals.

The preceding calculations are accurate only with certain line restrictions. The line propagation delay time must be longer than the rise time of the signal. If this condition is not met, the reflections will occur during the rise time and will not reach full amplitude. The above calculations also do not provide for capacitive loading. Reactive loading complicates the reflection coefficient by adding a time variable (Ref. 1, Chapt. 7). It is necessary to use a more sophisticated model to completely simulate MECL system interconnections. However, many systems can be adequately simulated by the use of purely resistive components.

| TIME | VOLTAGE AT B | TIME | VOLTAGE AT C |
|---|---|---|---|
| $T = 0$ | $B = V$ | $T = T_{PD}$ | $C = V + V(\rho_L)$ |
| $T = 2T_{PD}$ | $B = V + V(\rho_L) + V(\rho_L)(\rho_S)$ | $T = 3T_{PD}$ | $C = V + V(\rho_L) + V(\rho_L)(\rho_S) + V(\rho_L)^2(\rho_S)$ |
| $T = 0$ | B = 833 mV | $T = T_{PD}$ | C = 833 + 833 = 1666 mV |
| $T = 2T_{PD}$ | B = 833 + 833 - 555 = 1111 mV | $T = 3T_{PD}$ | C = 833 + 833 - 555 - 555 = 556 mV |

**FIGURE 2 — Reflection Calculations**

## THE COMPUTER MODEL

The circuit shown in Figure 3 was selected as a universal signal interconnection to be simulated on the computer. $C_T$ is the total capacitive load due to wiring capacitance and receiving circuit inputs. Although modeled as a lumped load at the end of the line, this capacitance can be used for the total load distributed along a line with little difference in the final result (a lumped load at the end is normally a worst case condition). $R_L$ is used in the circuit to provide for parallel termination and $R_S$ allows for series termination to improve waveforms. $R_o$ is the output impedance of the driving circuit and will be 7 to 10 ohms for MECL 10,000 or 5 to 8 ohms for MECL III.

No emitter pulldown resistor is used in the model for unterminated lines or series terminated lines. This resistor would be much larger than the output impedance of the driving circuit and would have little effect on $\rho_S$ in the calculations. When a pulldown resistor is required for circuit operation, it is assumed that the resistor will be sufficiently small to prevent the output transistor from turning off during a falling edge. Pulldown resistor considerations are discussed in Chapter 3 of the MECL System Design Handbook (Ref. 1).

Using Figure 3, the signal waveforms at the end of the line are derived in the following manner. $e_S$ is an internal ramp function voltage source in the driving device. The ramp slope is $E_1/T_1$ where $E_1$ is the open circuit steady state output and $T_1$ is the total slope rise time. Since logic circuits are commonly defined with 10 to 90% rise time $T_1 \approx 1.2\, t_r$. When calculating the slope of the signal at the transmission line input it is necessary to consider the line impedance, $Z_o$; the output impedance, $R_o$; and the termination resistors, $R_S$ and $R_L$.

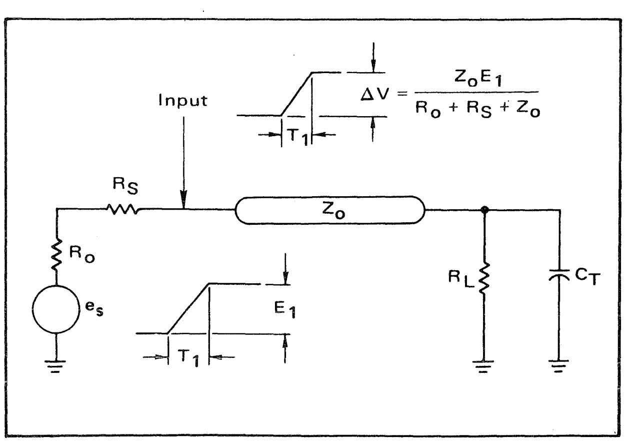



**FIGURE 3 — Transmission Line Model**

$$\text{Slope at Input} = m = \frac{Z_o E_1}{(R_o + R_S + Z_o)\, T_1}$$

Finally it is necessary to define the steady state voltage at the input to the line.

$$E_{SS} = \frac{E_1 R_L}{R_L + R_o + R_S}$$

The incident ramp signal on the line can be mathematically expressed during the ramp interval as:

$$e_i(t) = \frac{Z_o E_1 t}{(R_o + R_S + Z_o) T_1} \quad \text{for } t \leqslant T_1$$

and during the dc level after the ramp edge as:

$$e_i(t) = \frac{Z_o E_1}{(R_o + R_S + Z_o)} \quad \text{for } t > T_1$$

or

$$e_i(t) = mtU(t) - m(t - T_1)U(t - T_1)$$

$$\text{where } m = \text{slope} = \frac{Z_o E_1}{(R_o + R_S + Z_o)\, T_1}$$

Since the program determines the waveform at the load end of the line, t = time zero when the incident ramp signal reaches the load. The mathematical expression for the first reflection of a parallel loaded line has been derived in the MECL System Design Handbook, page 140, equation 29, [Ref. 1] and is:

$$e_{refl}(t) = -m \left[ \frac{2R_L^2 Z_o C_T}{(Z_o + R_L)^2} + \left( \frac{Z_o - R_L}{Z_o + R_L} \right) t - \left( \frac{2R_L^2 Z_o C_T}{(Z_o + R_L)^2} \right) e^{\frac{-(Z_o + R_L)t}{R_L Z_o C_T}} \right] U(t)$$

$$+ m \left[ \frac{2R_L^2 Z_o C_T}{(Z_o + R_L)^2} + \left( \frac{Z_o - R_L}{Z_o + R_L} \right) (t - T_1) - \left( \frac{2R_L^2 Z_o C_T}{(Z_o + R_L)^2} \right) e^{\frac{-(Z_o + R_L)(t - T_1)}{R_L Z_o C_T}} \right] U(t - T_1)$$

Since the amplitude at the end of the line after the first reflection is the sum of the initial signal on the line plus the first reflection, the output signal after the first reflection, $e_{out1}(t)$, is:

$$e_{out1}(t) = e_i(t) + e_{refl}(t)$$

$$e_{out1}(t) = \frac{2mR_L}{R_L + Z_o}\left[t - \frac{R_LZ_oC_T}{R_L + Z_o} + \left(\frac{R_LZ_oC_T}{R_L + Z_o}\right) e^{\frac{-(R_L + Z_o)t}{R_LZ_oC_T}}\right] U(t)$$

$$- \frac{2mR_L}{R_L + Z_o}\left[(t-T_1) - \frac{R_LZ_oC_T}{R_L + Z_o} + \left(\frac{R_LZ_oC_T}{R_L + Z_o}\right) e^{\frac{-(R_L + Z_o)(t-T_1)}{R_LZ_oC_T}}\right] U(t-T_1)$$

By solving the necessary equations it is possible to determine the output waveform for any required number of reflections. The equations get increasingly complex because of the reflection coefficients which must be considered. The following computer program uses the first nine reflections at the end of the line to determine the output signal. Nine reflections normally give sufficient time for the voltage to settle at approximately final amplitude. The appendix at the end of this application note shows the derivation of the equations used in the program to give the first nine reflections.

## USING THE COMPUTER PROGRAM

The program was written in BASIC on a time share terminal. After initializing the program, the following information is listed as being required. The title line identifies the program as a method of determining MECL system transmission line overshoot and undershoot.

```
MECL T-LINE OS AND US FOR OPEN AND PARALLEL TERMINATED LINES

RO= GATE OUTPUT IMPEDANCE IN OHMS
RS= SERIES DAMPING RESISTANCE IN OHMS
ZO= CHARACTERISTIC  IMPEDANCE OF T-LINE IN OHMS
RL= LOAD RESISTOR AT END OF LINE IN OHMS
CT= TOTAL LUMPED CAPACITANCE IN PICOFARADS
TD= PROP DELAY OF LINE IN NANOSECONDS/INCH
TR= RISE TIME OF GATE IN NANOSECONDS
L = LINE LENGTH IN INCHES
```

The following example uses a MECL 10,000 gate with 10 ohm output impedance and 3.5 ns rise time. MECL 10,000 circuits are specified with 2 ns 20 to 80% rise time, but the 3.5 ns is representative of the 10 to 90% edge speed required by the program. The circuit is driving an eight-inch, 50 ohm, unterminated microstrip line. No series damping resistor is used and the line is driving a test fixture having four MECL gate loads, an oscilloscope probe, and a probe holder. The total load capacitance is about 25 pF. Since no load or termination resistor is used, a large value of $R_L$ is inserted. 10,000 ohms is used to represent four MECL gate inputs. Propagation delay time of the line depends on the type of line used. 0.148 ns per inch is used for a microstrip line on a G-10 epoxy circuit board. The program will not operate with zero load capacitance. This causes a divide by zero calculation. Therefore, a very small $C_T$, for example 0.001 pF, should be used to simulate no capacitive load, if required.

The program then asks for the data inputs to be inserted after the question mark.

```
YOUR VALUES OF RO,RS,ZO,RL,CT, AND TR ARE
 ? 10,0,50,10000,25,3.5
WHAT IS TD AND L
 ? .148,8
TURN ON =          .91
RISE TIME =       2.45
OVERSHOOT(%)     TIME(NS)
 45.2             5.4
UNDERSHOOT(%)    TIME(NS)
 28.2             9.81
LAST TIME (NS)
 30.56
TO RECOMPUTE TYPE '1',   IF NOT TYPE '0'    ? 0
```

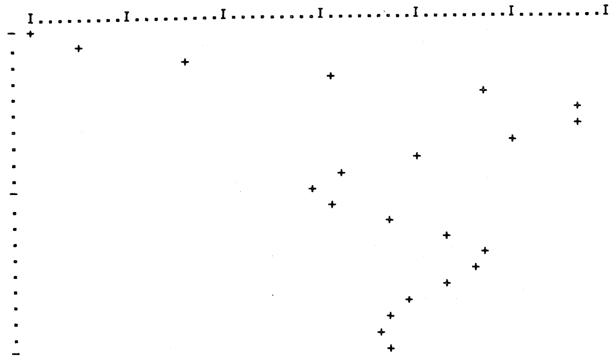
After receiving the data, calculations are made for turn-on time which is the 0 to 10% time in nanoseconds of the signal at the end of the line. The computed rise time of 2.45 ns is the 10 to 90% signal rise time. It is faster than the initial signal because of the large overshoot.

Overshoot is calculated as 45.2% of the final signal amplitude and occurs 5.4 ns after the start of the signal at the end of the line. The following undershoot is 28.2% and occurs at 9.81 ns after the start of the signal at the end of the line. One additional piece of information, LAST TIME, is given. Remembering that the program was developed using the first 9 reflections, there is a maximum time for which the calculated signal is valid. In this example it is 30.56 ns.

Following the computations, the program asks if another computation is to be performed or not. A zero (0) was typed after the question mark to allow the program to proceed to the plot routine. If a (1) were typed, the program would ask for a new set of variables.

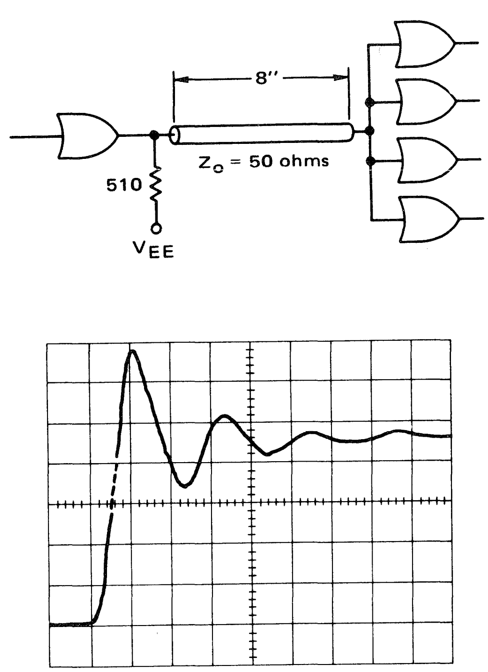
The plot routine is started by typing a 1 following the question mark. To establish a waveform it is necessary to set the vertical and horizontal coordinates. The amplitude is set by VMIN which is the starting point and VMAX which is the upper limit. The amplitude is broken into 60 divisions and VMAX defines the last one in terms of the final signal value. The voltage plot is normalized for a steady state voltage of 1.0. For the example with 45.2% overshoot VMAX should be at least 1.452 to keep the waveform on scale. The time base is defined by XMIN, the starting point, XMAX the maximum time of the plot and DELX the increment of time per plotted point in nanoseconds. XMAX should be less than LAST TIME to insure the plot remains within the maximum computer time. To plot the preceding calculations, the waveform is started at zero volts and zero time. The maximum amplitude is 1.5 which allows 50% overshoot and puts the final amplitude at the fourth major division. Time is plotted for 20 nanoseconds of 1 ns increments.

```
TO PLOT TYPE '1',  TO END PROGRAM TYPE '0'   ? 1

WHAT ARE VMIN,VMAX,XMIN,XMAX,DELX ? 0,1.5,0,20,1

FOR X:      TOP =  0  BOTTOM =  20  INCREMENT =  1
FOR V:      LEFT=  0  RIGHT  =  1.5  INCREMENT =  .025
```



```
TYPE '0' TO STOP OR '1' TO RECOMPUTE. WHICH ? 1
```

A test fixture was constructed to compare computed results with test results. Figure 4 shows the circuit under test and resulting waveforms. The correlation between the test oscilloscope waveform and the computer plot routine waveform is quite good.




Scale: Horizontal = 5 ns/div., Vertical = 200 mV/div.

**FIGURE 4 – MECL Test Circuit and Waveform**

Following the computer plot routine, the question is asked if a new computation will be performed of if the program is ended. Following a 1, new values are requested.

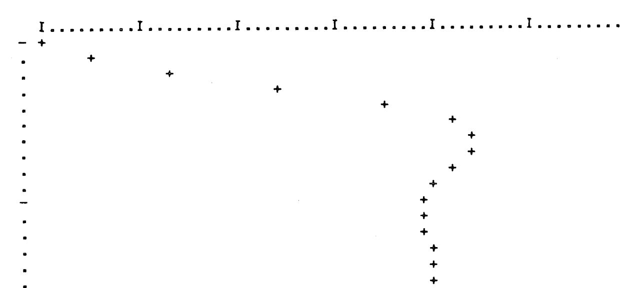
A second example uses the same line and loading as the previous example, but adds a 15 ohm series damping resistor to improve overshoot and undershoot.

```
YOUR VALUES OF RO,RS,ZO,RL,CT, AND TR ARE
 ? 10,15,50,10000,25,3.5
WHAT IS TD AND L
 ? .148,8
TURN ON =          1.05
RISE TIME =        2.94
OVERSHOOT(%)     TIME(NS)
 19.2              6.03
UNDERSHOOT(%)    TIME(NS)
 5.2               10.44
LAST TIME (NS)
 30.56
TO RECOMPUTE TYPE '1',  IF NOT TYPE '0'   ? 1
```

The computed information shows overshoot is improved from 45.2% to 19.2% and undershoot from 28.2% to 5.2% by adding the 15 ohm resistor. A badly distorted signal is improved to a very usable one by the small damping resistor.

A third example illustrates the value of parallel termination. The termination resistor $R_L$ is added to the test circuit used in the first example.

```
YOUR VALUES OF RO,RS,ZO,RL,CT, AND TR ARE
 ? 10,0,50,50,25,3.5
WHAT IS TD AND L
 ? .148,8
TURN ON =         .91
RISE TIME =       3.22
OVERSHOOT(%)    TIME(NS)
 11               6.51
UNDERSHOOT(%)   TIME(NS)
 1.7              10.58
LAST TIME (NS)
 30.56
TO RECOMPUTE TYPE '1',  IF NOT TYPE '0'   ? 0

TO PLOT TYPE '1',  TO END PROGRAM TYPE '0'   ? 1

WHAT ARE VMIN,VMAX,XMIN,XMAX,DELX ? 0,1.5,0,15,1

FOR X:      TOP =  0  BOTTOM =  15  INCREMENT =  1
FOR V:      LEFT=  0  RIGHT  =  1.5  INCREMENT =  .025
```



```
TYPE '0' TO STOP OR '1' TO RECOMPUTE. WHICH ? 0
```

The 50 ohm termination resistor reduced overshoot to 11% and undershoot to 1.7%. The small overshoot is due to the impedance mismatch caused by the capacitive load. The plot shows the waveform improvement over the unterminated line in the first example.

The zero after the question mark ends the program.

## CONCLUSION

This program has been written for a high degree of flexibility. It can be used with a large variety of line types and driving sources. However, it is possible to modify the program to provide more specialized information. For example, tables can be written for a predetermined overshoot or undershoot and specified line types. Tables of this type are found in Chapter 3 of the MECL System Design Handbook for 35% overshoot or 12% undershoot. Tables can be generated to develop system wiring rules for a selected amount of signal distortion. Other possible modifications to the program include plotting actual voltage output instead of normalized voltage and calculations of line delays to help determine system delay times.

The program was designed to be used for a MECL system. MECL circuits have low output impedance in both logic states, the ability to drive terminated transmission lines, high input impedance, and nearly symmetrical noise margins for both high and low logic levels. However, the program is adaptable to other logic types if the input and output impedances are known. For example, the program was used to determine the waveform of a MOS memory clock driver required to drive a high capacitive load.

The relatively slow rise and fall times of MECL 10,000 circuits, typically 3.5 ns 10-90%, do not require a strict transmission line environment or completely modeled interconnections for every line in the system. It is expected this technique of using a computer program to determine waveshapes will be most useful in deciding if a marginally long line should be terminated or to help establish proper termination for a heavily loaded line. Testing has shown a very good correlation between computed results and test results in the laboratory. The designer building a MECL system will find that many of the system layout considerations can be solved with the aid of computer terminal.

## THE COMPUTER PROGRAM

```
10 PRINT "MECL T-LINE OS AND US FOR OPEN AND PARALLEL TERMINATED LINES"
11 REM ************************************************************
12 REM COPYRIGHT 1972, MOTOROLA, INC.
13 REM ************************************************************
15 PRINT
20 PRINT "RO= GATE OUTPUT IMPEDANCE IN OHMS"
30 PRINT "RS= SERIES DAMPING RESISTANCE IN OHMS"
40 PRINT "ZO= CHARACTERISTIC  IMPEDANCE OF T-LINE IN OHMS"
45 PRINT "RL= LOAD RESISTOR AT END OF LINE IN OHMS"
50 PRINT "CT= TOTAL LUMPED CAPACITANCE IN PICOFARADS"
60 PRINT "TD= PROP DELAY OF LINE IN NANOSECONDS/INCH"
70 PRINT "TR= RISE TIME OF GATE IN NANOSECONDS"
72 PRINT "L = LINE LENGTH IN INCHES"
75 PRINT
80 PRINT "YOUR VALUES OF RO,RS,ZO,RL,CT, AND TR ARE"
90 INPUT R0,R1,Z0,R2,C1,T0
91 PRINT "WHAT IS TD AND L"
92 INPUT T4,L
93 LET T3=T4*L
94 LET C=.001*C1
95 LET E1=R2/(R0+R1+R2)
96 LET A=(R2-Z0)/(R2*Z0*C)
97 LET B=(R2+Z0)/(R2*Z0*C)
98 LET M=Z0/(1.2*T0*(R1+R0+Z0))
99 LET P=(R0+R1-Z0)/(R0+R1+Z0)
100 LET A1=2*M/(Z0*C*B↑2)
101 LET A2=A1*P
102 LET A3=A2*P
103 LET A4=A3*P
104 LET A5=A4*P
105 LET A6=A5*P
106 LET A7=A6*P
107 LET B1=A/B
108 LET B2=(A/B)↑2
109 LET B3=(A/B)↑3
110 LET B4=(A/B)↑4
111 LET B5=(A/B)↑5
112 LET B6=(A/B)↑6
113 LET C0=T3*1000/Z0
114 LET F9=SQR(1+C1/C0)
115 LET Y=18*T3*F9
116 LET A8=A7*P
117 LET A9=A8*P
118 LET B7=(A/B)↑7
119 LET B8=(A/B)↑8
150 LET X=0
155 READ V8,V9,D,F,G0,G1
160 DATA 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
170 GOSUB 970
350 IF G0=1 THEN 375
355 IF V8<.1 THEN 475
360 PRINT "TURN ON = ",(INT(100*X+.5))/100
362 LET G2=X
365 LET G0=1
370 GO TO 475
372 LET X=X+.5*T0
375 IF G1=1 THEN 385
377 IF V8<.9 THEN 475
379 PRINT "RISE TIME = ",(INT(100*(X-G2)+.5))/100
381 LET G1=1
383 GO TO 480
385 IF D=1 THEN 438
390 IF V8>V9 THEN 480
400 PRINT "OVERSHOOT(%)","TIME(NS)"
410 PRINT (INT((V9-1)*1000+.5))/10,(INT(100*W+.5))/100
420 LET D=1
422 LET W=X
424 LET V9=V8
430 LET X=1.2*T3*F9+X
435 GO TO 170
438 IF F=1 THEN 480
440 IF V8<V9 THEN 480
450 PRINT"UNDERSHOOT(%)","TIME(NS)"
460 PRINT (INT((1-V9)*1000+.5))/10,(INT(100*W+.5))/100
461 LET F=1
462 LET W=X
464 LET V9=V8
465 LET X=Y
470 GO TO 520
475 LET X=X+.02*T0
477 GO TO 170
480 LET W=X
485 LET V9=V8
490 IF X>=Y THEN 520
500 LET X=X+.2*T3*F9
510 GO TO 170
520 PRINT "LAST TIME (NS)"
530 PRINT (INT(100*X+.5))/100
540 PRINT "TO RECOMPUTE TYPE '1',  IF NOT TYPE '0'  ";
550 INPUT H
560 RESTORE
561 PRINT
570 IF H=1 THEN 80
580 PRINT "TO PLOT TYPE '1',  TO END PROGRAM TYPE '0'  ";
590 INPUT H1
600 IF H1=1 THEN 903
610 GO TO 1854
903 PRINT
904 PRINT "WHAT ARE VMIN,VMAX,XMIN,XMAX,DELX";
905 INPUT Q0,Q1,Q2,Q3,Q4
906 LET Q5=(Q1-Q0)/60
907 LET Q6=0
909 FOR X =Q2 TO Q3 STEP Q4
910 GOSUB 970
911 IF Q6=0 THEN 937
912 IF Q6=20 THEN 915
913 PRINT " . ";
914 GO TO 917
915 PRINT " - ";
916 LET Q6=10
917 IF V>Q1 THEN 934
918 IF V<Q0 THEN 934
919 LET Q7=Q0+2*Q5
920 LET Z=Q7+0.5*Q5
921 IF Z<V THEN 931
922 LET Q6 =Q6+1
923 IF Z-V>=2*Q5 THEN 929
924 IF Z-V>=Q5 THEN 927
925 PRINT "  +"
926 GO TO 943
927 PRINT " +"
928 GO TO 943
929 PRINT "+"
930 GO TO 943
931 LET Q7=Q7+3*Q5
932 PRINT "   ";
933 GO TO 920
934 PRINT "OFF SCALE"
935 LET Q6=Q6+1
936 GO TO  943
937 PRINT
938 PRINT "FOR X:     TOP = ";Q2;" BOTTOM = ";Q3;" INCREMENT = ";Q4
939 PRINT "FOR V:     LEFT= ";Q0;" RIGHT  = ";Q1;" INCREMENT = ";Q5
940 PRINT
941 PRINT "   I.........I.........I.........I.........I.........I.........I"
942 GO TO 915
943 NEXT X
944 PRINT
945 PRINT "TYPE '0' TO STOP OR '1' TO RECOMPUTE. WHICH";
946 INPUT Q8
947 IF Q8=0 THEN 1854
948 IF Q8=1 THEN 80
949 PRINT
950 GO TO 944
970 LET E=2.71828
972 READ V1,V2,V3,V4,V5,V6,V7,N0,N1,N2,N3,N4,N5,N6,N7,N8,N9
975 LET V0=A1*(E↑(-B*X)+B*X-1)
977 LET T1=1.2*T0
979 IF X<T1 THEN 983
981 LET V1=A1*(E↑((T1-X)*B)+B*(X-T1)-1)
983 LET T=2*T3
985 IF X<T THEN 989
987 LET V2=A2*(-2*B1-1+A*(X-T)+(2*B1+1+(A+B)*(X-T))*E↑((T-X)*B))
989 LET T=2*T3+T1
991 IF X<T THEN 995
993 LET V3=A2*(-2*B1-1+A*(X-T)+(2*B1+1+(A+B)*(X-T))*E↑((T-X)*B))
995 LET T=4*T3
997 IF X<T THEN 1004
999 LET N=X-T
1000 LET V4=A3*B1*(-3*B1-2+A*N+(3*B1+2+2*(A+B)*N)*E↑(-B*N))
1002 LET V4=V4+A3*(B1+B2/2+.5)*(B*N)↑2*E↑(-B*N)
1004 LET T=4*T3+T1
1006 IF X<T THEN 1014
1008 LET N=X-T
1010 LET V5=A3*B1*(-3*B1-2+A*N+(3*B1+2+2*(A+B)*N)*E↑(-B*N))
1012 LET V5=V5+A3*(B1+B2/2+.5)*(B*N)↑2*E↑(-B*N)
1014 LET T=6*T3
1016 IF X<T THEN 1026
1018 LET N=X-T
1020 LET V6=A4*B2*(-4*B1-3+A*N+(4*B1+3+3*(A+B)*N)*E↑(-B*N))
1022 LET V6=V6+A4*(1.5*B2+B3-.5)*(B*N)↑2*E↑(-B*N)
1024 LET V6=V6+A4*(B2/2+B3/6+B1/2+1/6)*(B*N)↑3*E↑(-B*N)
1026 LET T=6*T3+T1
1028 IF X<T THEN 1038
1030 LET N=X-T
1032 LET V7=A4*B2*(-4*B1-3+A*N+(4*B1+3+3*(A+B)*N)*E↑(-B*N))
1034 LET V7=V7+A4*(1.5*B2+B3-.5)*(B*N)↑2*E↑(-B*N)
1036 LET V7=V7+A4*(B2/2+B3/6+B1/2+1/6)*(B*N)↑3*E↑(-B*N)
1038 LET T=8*T3
1040 IF X<T THEN 1050
1042 LET N=X-T
1044 LET N0=A5*B3*(-5*B1-4+A*N+(5*B1+4+4*(A+B)*N)*E↑(-B*N))
1046 LET N0=N0+A5*(2*B3+1.5*B4+.5)*(B*N)↑2*E↑(-B*N)
1047 LET N0=N0+A5*(2*B3/3+B4/3-2*B1/3-1/3)*(B*N)↑3*E↑(-B*N)
1048 LET N0=N0+A5*(B3/6+B4/24+B2/4+B1/6+1/24)*(B*N)↑4*E↑(-B*N)
1050 LET T=8*T3+T1
1052 IF X<T THEN 1062
1054 LET N=X-T
1056 LET N1=A5*B3*(-5*B1-4+A*N+(5*B1+4+4*(A+B)*N)*E↑(-B*N))
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1058 LET N1=N1+A5*(2*B3+1.5*B4+.5)*(B*N)↑2*E↑(-B*N)
1059 LET N1=N1+A5*(2*B3/3+B4/3-2*B1/3-1/3)*(B*N)↑3*E↑(-B*N)
1060 LET N1=N1+A5*(B3/6+B4/24+B2/4+B1/6+1/24)*(B*N)↑4*E↑(-B*N)
1062 LET T=10*T3
1064 IF X<T THEN 1080
1066 LET N=X-T
1068 LET U=(X-T)*B
1070 LET N2=A6*B4*(-6*B1-5+A*N+(6*B1+5+5*(A+B)*N+(5/2+2*B1)*U↑2)*E↑(-U))
1072 LET N2=N2+A6*(-.5*U↑2+(B4*5/6+B5/2+B1*5/6+.5)*U↑3)*E↑(-U)
1074 LET N2=N2+A6*(B4*5/24+B5/12-B2*5/12-B1*5/12-1/8)*U↑4*E↑(-U)
1076 LET N2=N2+A6*(B4*5/120+B5/120+B3/12+B2/12+B1/24+1/120)*U↑5*E↑(-U)
1080 LET T=10*T3+T1
1082 IF X<T THEN 1098
1084 LET N=X-T
1086 LET U=(X-T)*B
1088 LET N3=A6*B4*(-6*B1-5+A*N+(6*B1+5+5*(A+B)*N+(5/2+2*B1)*U↑2)*E↑(-U))
1090 LET N3=N3+A6*(-.5*U↑2+(B4*5/6+B5/2+B1*5/6+.5)*U↑3)*E↑(-U)
1092 LET N3=N3+A6*(B4*5/24+B5/12-B2*5/12-B1*5/12-1/8)*U↑4*E↑(-U)
1094 LET N3=N3+A6*(B4*5/120+B5/120+B3/12+B2/12+B1/24+1/120)*U↑5*E↑(-U)
1098 LET T=12*T3
1100 IF X<T THEN 1118
1102 LET N=X-T
1104 LET U=(X-T)*B
1106 LET N4=A7*B5*(-7*B1-6+A*N+(7*B1+6+6*(A+B)*N+(3+B1*5/2)*U↑2)*E↑(-U))
1108 LET N4=N4+A7*(.5*U↑2+(B5+B6*2/3-B1-2/3)*U↑3)*E↑(-U)
1110 LET N4=N4+A7*(B5/4+B6/8+B2*5/8+B1*3/4+.25)*U↑4*E↑(-U)
1112 LET N4=N4+A7*(B5/20+B6/60-B3/6-B2/4-B1*3/20-1/30)*U↑5*E↑(-U)
1114 LET N4=N4+A7*(B5/120+B6/720+B4/48+B3/36+B2/48)*U↑6*E↑(-U)
1116 LET N4=N4+A7*(B1/120+1/720)*U↑6*E↑(-U)
1118 LET T=12*T3+T1
1120 IF X<T THEN 1138
1122 LET N=X-T
1124 LET U=(X-T)*B
1126 LET N5 =A7*B5*(-7*B1-6+A*N+(7*B1+6+6*(A+B)*N+(3+B1*5/2)*U↑2)*E↑(-U))
1128 LET N5=N5+A7*(.5*U↑2+(B5+B6*2/3-B1-2/3)*U↑3)*E↑(-U)
1130 LET N5=N5+A7*(B5/4+B6/8+B2*5/8+B1*3/4+.25)*U↑4*E↑(-U)
1132 LET N5=N5+A7*(B5/20+B6/60-B3/6-B2/4-B1*3/20-1/30)*U↑5*E↑(-U)
1134 LET N5=N5+A7*(B5/120+B6/720+B4/48+B3/36+B2/48)*U↑6*E↑(-U)
1136 LET N5=N5+A7*(B1/120+1/720)*U↑6*E↑(-U)
1138 LET T=14*T3
1140 IF X<T THEN 1162
1142 LET N=X-T
1144 LET U=(X-T)*B
1146 LET N6=A8*B6*(-8*B1-7+A*N+(8*B1+7+7*(A+B)*N)*E↑(-U))
1148 LET N6=N6+A8*((B6*7/2+B7*3-.5)*U↑2+(B6*7/6+B7*5/6)*U↑3)*E↑(-U)
1150 LET N6=N6+A8*((B1*7/6+5/6)*U↑3+(B6*7/24+B7/6-B2*7/8)*U↑4)*E↑(-U)
1152 LET N6=N6+A8*((-B1*7/6-5/12)*U↑4+(B6*7/120+B7/40)*U↑5)*E↑(-U)
1154 LET N6=N6+A8*(B3*7/24+B2*21/40+B1*7/20+1/12)*U↑5*E↑(-U)
1156 LET N6=N6+A8*(B6*7/720+B7/360-B4*7/144-B3*7/72-B2*7/80)*U↑6*E↑(-U)
1158 LET N6=N6+A8*((-B1*7/180-1/144)*U↑6+(B6/720+B7/5040)*U↑7)*E↑(-U)
1160 LETN6=N6+A8*(B5/240+B4/144+B3/144+B2/240+B1/720+1/5040)*U↑7*E↑(-U)
1162 LET T=14*T3+T1
1164 IF X<T THEN 1186
1166 LET N=X-T
1168 LET U=(X-T)*B
1170 LET N7=A8*B6*(-8*B1-7+A*N+(8*B1+7+7*(A+B)*N)*E↑(-U))
1172 LET N7=N7+A8*((B6*7/2+B7*3-.5)*U↑2+(B6*7/6+B7*5/6)*U↑3)*E↑(-U)
1174 LET N7=N7+A8*((B1*7/6+5/6)*U↑3+(B6*7/24+B7/6-B2*7/8)*U↑4)*E↑(-U)
1176 LET N7=N7+A8*((-B1*7/6-5/12)*U↑4+(B6*7/120+B7/40)*U↑5)*E↑(-U)
1178 LET N7=N7+A8*(B3*7/24+B2*21/40+B1*7/20+1/12)*U↑5*E↑(-U)
1180 LET N7=N7+A8*(B6*7/720+B7/360-B4*7/144-B3*7/72-B2*7/80)*U↑6*E↑(-U)
1182 LET N7=N7+A8*((-B1*7/180-1/144)*U↑6+(B6/720+B7/5040)*U↑7)*E↑(-U)
1184 LET N7=N7+A8*(B5/240+B4/144+B3/144+B2/240+B1/720+1/5040)*U↑7*E↑(-U)
1186 LET T=16*T3
1188 IF X<T THEN 1212
1190 LET N=X-T
1192 LET U=(X-T)*B
1194 LET N8=A9*B7*(-9*B1-8+A*N+(9*B1+8+8*(A+B)*N)*E↑(-U))
1196 LET N8=N8+A9*((B7*4+B8*3.5+.5)*U↑2+(B7*4/3+B8-B1*4/3-1)*U↑3)*E↑(-U)
1198 LET N8=N8+A9*(B7/3+B8*5/24+B2*35/30+B1*5/3+5/8)*U↑4*E↑(-U)
1200 LET N8=N8+A9*(B7/15+B8/30-B3*7/15-B2*14/15-B1*2/3-1/6)*U↑5*E↑(-U)
1202 LET N8=N8+A9*(B7/90+B8/240+B4*7/72+B3*7/30+B2*7/30+B1/9)*U↑6*E↑(-U)
1204 LET N8=N8+A9*(U↑6/48+(B7/630+B8/2520-B5/90-B4/36-B3/30)*U↑7)*E↑(-U)
1206 LET N8=N8+A9*((-B2/45-B1/126-1/840)*U↑7+B7*U↑8/5040)*E↑(-U)
1208 LET N8=N8+A9*(B8/40320+B6/1440+B5/720+B4/576)*U↑8*E↑(-U)
1210 LET N8=N8+A9*(B3/720+B2/1440+B1/5040+1/40320)*U↑8*E↑(-U)
1212 LET T=16*T3+T1
1214 IF X<T THEN 1800
1216 LET N=X-T
1218 LET U=(X-T)*B
1220 LET N9=A9*B7*(-9*B1-8+A*N+(9*B1+8+8*(A+B)*N)*E↑(-U))
1222 LET N9=N9+A9*((B7*4+B8*3.5+.5)*U↑2+(B7*4/3+B8-B1*4/3-1)*U↑3)*E↑(-U)
1224 LET N9=N9+A9*(B7/3+B8*5/24+B2*35/30+B1*5/3+5/8)*U↑4*E↑(-U)
1226 LET N9=N9+A9*(B7/15+B8/30-B3*7/15-B2*14/15-B1*2/3-1/6)*U↑5*E↑(-U)
1228 LET N9=N9+A9*(B7/90+B8/240+B4*7/72+B3*7/30+B2*7/30+B1/9)*U↑6*E↑(-U)
1230 LET N9=N9+A9*(U↑6/48+(B7/630+B8/2520-B5/90-B4/36-B3/30)*U↑7)*E↑(-U)
1232 LET N9=N9+A9*((-B2/45-B1/126-1/840)*U↑7+B7*U↑8/5040)*E↑(-U)
1234 LET N9=N9+A9*(B8/40320+B6/1440+B5/720+B4/576)*U↑8*E↑(-U)
1236 LET N9=N9+A9*(B3/720+B2/1440+B1/5040+1/40320)*U↑8*E↑(-U)
1800 LET V8=V0-V1+V2-V3+V4-V5+V6-V7+N0-N1+N2-N3+N4-N5+N6-N7+N8-N9
1805 LET V8=V8/E1
1810 LET V=V8
1850 RESTORE
1852 RETURN
1854 END
```

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## APPENDIX

The following is a derivation of the equations used to write the computer program in the preceding section. Figure 3 illustrates the transmission line model used for the equations.

The load at the end of the line, expressed in LaPlace notation is:

$$Z_L = \frac{R_L\left(\frac{1}{SC_T}\right)}{R_L + \frac{1}{SC_T}} = \frac{R_L}{SR_LC_T + 1}$$

The reflection coefficient at the load end of the line is:

$$\rho_L(S) = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{-\left(S - \frac{R_L - Z_o}{R_LZ_oC_T}\right)}{S + \frac{R_L + Z_o}{R_LZ_oC_T}}$$

and

$$1 + \rho_L(S) = \frac{\frac{2}{Z_oC_T}}{S + \frac{R_L + Z_o}{R_LZ_oC_T}}$$

The incident signal on the line is

$$e_i(t) = mtU(t) - m(t-T_1)\,U(t-T_1)$$

where $m = \text{slope} = \dfrac{Z_oE_1}{(R_o + R_S + Z_o)\,T_1}$

Taking the LaPlace transform of $e_i(t)$ results in

$$E_i(S) = \frac{m}{S^2}\left(1 - e^{-T_1S}\right)$$

The signal at the load end of the line due to the incident voltage is:

$$E_{o1}(S) = E_i(S) + E_{REF\ 1}(S) = E_i(S)\,[1 + \rho_L(S)]$$

$$E_{o1}(S) = \frac{\frac{2m}{Z_oC_T}\left(1 - e^{-T_1S}\right)}{S^2\left(S + \frac{R_L + Z_o}{R_LZ_oC_T}\right)}$$

The second reflected voltage waveform at the load occurs at a time 2 line delays later, $2T_D$, and can be written as:

$$E_{o2}(S) = E_i(S)\,\rho_S \rho_L(S)\,[1 + \rho_L(S)]\,e^{-2T_D S}$$

$$= \frac{-2m\rho_S}{Z_o C_T}\left[\frac{\left(S - \frac{R_L - Z_o}{R_L Z_o C_T}\right)}{S^2\left(S + \frac{R_L + Z_o}{R_L Z_o C_T}\right)^2}\,(1 - e^{-T_1 S})\,e^{-2T_D S}\right]$$

The third reflected voltage waveform at the load is:

$$E_{o3}(S) = E_i(S)\,\rho_S{}^2 \rho_L(S)^2\,[1 + \rho_L(S)]\,e^{-4T_D S}$$

$$= \frac{2m\rho_S{}^2}{Z_o C_T}\left[\frac{\left(S - \frac{R_L - Z_o}{R_L Z_o C_T}\right)^2}{S^2\left(S + \frac{R_L + Z_o}{R_L Z_o C_T}\right)^3}\,(1 - e^{-T_1 S})\,e^{-4T_D S}\right]$$

Beyond the third reflection the LaPlace equations become more difficult to solve and it is desirable to have a general equation which may be used for any number, n, reflections.

$$E_{on}(S) = E_i(S)\rho_S{}^{n-1}\rho_L(S)^{n-1}\,[1 + \rho_L(S)]\;e^{-2(n-1)T_D S}$$

$$= \frac{(-1)^{n-1}2m\rho_S{}^{n-1}}{Z_o C_T}\left[\frac{\left(S - \frac{R_L - Z_o}{R_L Z_o C_T}\right)^{n-1}}{S^2\left(S + \frac{R_L + Z_o}{R_L Z_o C_T}\right)^n}\right]$$

$$\left[(1 - e^{-T_1 S})e^{-2(n-1)T_D S}\right]$$

Letting $\frac{R_L - Z_o}{R_L Z_o C_T} = a$ and $\frac{R_L + Z_o}{R_L Z_o C_T} = b$

The LaPlace transform of $E_{01}(S)$ is of the form:

$$F_1(S) = \frac{1}{S^2(S + b)}$$

Solving the inverse LaPlace transform gives:

$$F_1(t) = \frac{1}{b^2}(-1 + bt + e^{-bt})$$

The equation for the voltage at the end of the line due to the incident signal can then be written:

$$e_{01}(t) = \frac{-2m}{Z_o C_T b^2}\,(-1 + bt + e^{-bt})U(t)$$

$$-\frac{-2m}{Z_o C_T b^2}\,[-1 + b(t - T_1) + e^{-b(t-T_1)}]\;U(t - T_1)$$

The LaPlace transform of $e_{on}(t)$ has the form:

$$F_n(S) = \frac{(S - a)^{n-1}}{S^2(S + b)^n} = \frac{(-1)^{n-1}a^{n-1}\left(1 - \frac{S}{a}\right)^{n-1}}{b^n S^2\left(1 + \frac{S}{b}\right)^n} \qquad (1a)$$

The binomial expansion can be used in the numerator to obtain:

$$F_n(S) = \frac{(-1)^{n-1}a^{n-1}}{b^n S^2\left(1 + \frac{S}{b}\right)^n}\sum_{i=0}^{n-1}(-1)^i\binom{n-1}{i}\left(\frac{1}{a}\right)^i S^i \qquad (1b)$$

where $\binom{X}{Y} = \frac{X!}{Y!\,(X-Y)!}$

Equation (1b) can be expanded by separating and rearranging the first two terms of the summation to obtain.

$$F_n(S) = \frac{(-1)^{n-1}a^{n-1}}{b^n}\left[\frac{1}{S^2\left(1 + \frac{S}{b}\right)^n} - \frac{(n-1)}{a}\,\frac{1}{S\left(1 + \frac{S}{b}\right)^n}\right.$$

$$\left. + \sum_{i=2}^{n-1}(-1)^i\,\frac{\binom{n-1}{i}\left(\frac{1}{a}\right)^i S^{i-2}}{\left(1 + \frac{S}{b}\right)^n}\right] \qquad (2)$$

the inverse LaPlace transform of the last two terms in the above equation can be found in Reference 4 which can be written

$$\mathscr{L}^{-1}F_2(S) = \mathscr{L}^{-1}\frac{1}{S\left(1 + \frac{S}{b}\right)^n} = 1 - e^{-bt}\sum_{i=0}^{n-1}\frac{(bt)^i}{i!} \qquad (3)$$

$$\mathscr{L}^{-1}F_3(S) = \mathscr{L}^{-1}\frac{S^{i-2}}{\left(1 + \frac{S}{b}\right)^n} = b^{i-1}\,e^{-bt}$$

$$\sum_{j=0}^{i-2}(-1)^{i-2-j}\binom{i-2}{j}\frac{(bt)^{n-j-1}}{(n-j-1)!} \qquad (4)$$

The first sterm of equation (2) can be solved by using the general property

$$\mathscr{L}^{-1}\frac{1}{S}F_n(S) = \int_0^t f(t)dt \qquad (5)$$

Therefore

$$\mathscr{L}^{-1}F_1(S) = \mathscr{L}^{-1}\frac{1}{S^2\left(1 + \frac{S}{b}\right)^n} = \mathscr{L}^{-1}\frac{1}{S}F_2(S) =$$

$$\int_0^t\left(1 - e^{-bt}\sum_{i=0}^{n-1}\frac{(bt)^i}{i!}\right)dt =$$

$$t - \frac{1}{b}\sum_{i=0}^{n-1}\frac{1}{i!}\int_0^t e^{-bt}(bt)^i\,b\,dt \qquad (6)$$

The integral of equation (6) has been evaluated in Reference 4. Therefore equation (6) can be written

$$\mathcal{L}^{-1} F_1(S) = t - \frac{1}{b}\sum_{i=0}^{n-1}\left[1 - e^{-bt}\sum_{j=0}^{i}\frac{(bt)^j}{j!}\right] \qquad (7)$$

Using equations 2, 3, 4 and 7 the inverse transform of equation 1a can be written

$$\mathcal{L}^{-1} F_n(S) = \frac{(-1)^{n-1}a^{n-1}}{b^n}\left\{\frac{-(n-1)}{a}\left[1-e^{-bt}\sum_{i=0}^{n-1}\frac{(bt)^i}{i!}\right] + \sum_{i=2}^{n-1}(-1)^i\binom{n-1}{i}\left(\frac{1}{a}\right)^i b^{i-1}e^{-bt}\sum_{j=0}^{i-2}\right.$$

$$\left.(-1)^{i-2-j}\binom{i-2}{j}\frac{(bt)^{n-j-1}}{(n-j-i)!} + t - \frac{1}{b}\sum_{i=0}^{n-1}\left[1 - e^{-bt}\sum_{j=0}^{i}\frac{(bt)^j}{j!}\right]\right\} \qquad (8)$$

This equation can be rearranged to

$$\mathcal{L}^{-1} F_n(S) = (-1)^{n-1}\frac{a^{n-2}}{b^n}\left\{-n\left(\frac{a}{b}\right) - n + 1 + at + \left[n\left(\frac{a}{b}\right) + n - 1\right]e^{-bt}\right\} + \frac{(-1)^{n-1}}{b^2}e^{-bt}\left[\left(\frac{a}{b}\right)^{n-2}(n-1)\right.$$

$$\sum_{i=1}^{n-1}\frac{(bt)^i}{i!} + \left(\frac{a}{b}\right)^{n-1}\sum_{i=1}^{n-1}\sum_{j=1}^{i}\frac{(bt)^j}{j!} +$$

$$\left.\sum_{i=2}^{n-1}\sum_{j=0}^{i-2}\left(\frac{a}{b}\right)^{n-1-i}(-1)^j\binom{n-1}{i}\binom{i-2}{j}\frac{(bt)^{n-j-1}}{(n-j-1)!}\right]$$

The voltage waveform due to the nth reflection can now be written as:

$$e_{on}(t) = \frac{2m\rho s^{n-1}}{Z_oC_T}\left\{\frac{a^{n-2}}{b^n}\left[\frac{-na}{b} - n+1+a[t-2(n-1)T_D]\right.\right.$$

$$\left. + \left(\frac{na}{b} + n-1\right)e^{-b[t-2(n-1)T_D]}\right] + \frac{e^{-b[t-2(n-1)T_D]}}{b^2}$$

$$\left[\left(\frac{a}{b}\right)^{n-2}(n-1)\sum_{i=1}^{n-1}\frac{b[t-2(n-1)T_D]^i}{i!} + \right.$$

$$\left(\frac{a}{b}\right)^{n-1}\sum_{i=1}^{n-1}\sum_{j=i}^{i}\frac{\{b[t-2(n-1)T_D]\}^j}{j!} +$$

$$\sum_{i=2}^{n-1}\sum_{j=0}^{i-2}\left(\frac{a}{b}\right)^{n-1-i}(-1)^j\left(\frac{(n-1)!}{i!\,(n-1-i)!}\right)\left(\frac{(i-2)!}{j!\,(i-2-j)!}\right)$$

$$\left.\left.\frac{\{b[t-2(n-1)T_D]\}^{n-j-1}}{(n-j-1)!}\right]\right\} U[t-2(n-1)T_D]$$

$$-\frac{2m\rho s^{n-1}}{Z_oC_T}\left\{\frac{a^{n-2}}{b^2}\left[\frac{-na}{b} - n+1+a(t-T_1-2(n-1)T_D)\right.\right.$$

$$\left. + \left(\frac{na}{b} + n-1\right)e^{-b[t-T_1-2(n-1)T_D]}\right]$$

$$+ \frac{e^{-b[t-T_1-2(n-1)T_D]}}{b^2}\left[\left(\frac{a}{b}\right)^{n-2}(n-1)\sum_{i=1}^{n-1}\right.$$

$$\frac{\{b[t-T_1-2(n-1)T_D]\}^i}{i!} + \left(\frac{a}{b}\right)^{n-1}\sum_{i=1}^{n-1}\sum_{j=1}^{i}$$

$$\frac{\{b[t-T_1-2(n-1)T_D]\}^j}{j!} + \sum_{i=2}^{n-1}\sum_{j=0}^{i=2}\left(\frac{a}{b}\right)^{n-1-i}(-1)^j$$

$$\left.\left.\left(\frac{(n-1)!}{i!(n-1-i)!}\right)\left(\frac{(i-2)!}{j!(i-2-j)!}\right)\frac{\{b[t-T_1-2(n-1)T_D]\}^{n-j-1}}{(n-j-1)!}\right]\right\}$$

$$U[t-T_1-2(n-1)T_D]$$

The output voltage at the end of the line is the sum of the incident signal and the reflections.

$$e_{out}(t) = \sum_{n=1}^{n} e_{on}(t)$$

The following is a list of the equations for F(t) as used in the computer program.

$$F_1(t) = \frac{1}{b^2}\left[e^{-bt} + b^t - 1\right]$$

$$F_2(t) = \frac{-1}{b^2}\left\{\frac{-2a}{b} - 1 + at + \left[\frac{2a}{b} + 1 + (a+b)t\right] e^{-bt}\right\}$$

$$F_3(t) = \frac{1}{b^2}\left\{\frac{a}{b}\left[\frac{-3a}{b} - 2 + at + [\frac{3a}{b} + 2 + 2(a+b)t]\, e^{-bt}\right]\right.$$
$$\left. + \left[\frac{a}{b} + \frac{1}{2}\left(\frac{a}{b}\right)^2 + \frac{1}{2}\right](bt)^2 e^{-bt}\right\}$$

$$F_4(t) = \frac{-1}{b^2}\left\{\left(\frac{a}{b}\right)^2\left[\frac{-4a}{b} - 3 + at + [\frac{4a}{b} + 3 + 3(a+b)t]\, e^{-bt}\right]\right.$$
$$+ \left[\frac{3}{2}\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right)^3 - \frac{1}{2}\right](bt)^2 e^{-bt}$$
$$\left. + \left[\frac{1}{2}\left(\frac{a}{b}\right)^2 + \frac{1}{6}\left(\frac{a}{b}\right)^3 + \frac{1}{2}\left(\frac{a}{b}\right) + \frac{1}{6}\right](bt)^3 e^{-bt}\right\}$$

$$F_5(t) = \frac{1}{b^2}\left\{\left(\frac{a}{b}\right)^3\left[\frac{-5a}{b} - 4 + at + [\frac{-5a}{b} + 4 + 4(a+b)t]\, e^{-bt}\right]\right.$$
$$+ \left[2\left(\frac{a}{b}\right)^3 + \frac{3}{2}\left(\frac{a}{b}\right)^4 + \frac{1}{2}\right](bt)^2 e^{-bt}$$
$$+ \left[\frac{2}{3}\left(\frac{a}{b}\right)^3 + \frac{1}{3}\left(\frac{a}{b}\right)^4 - \frac{2}{3}\left(\frac{a}{b}\right) - \frac{1}{3}\right](bt)^3 e^{-bt}$$
$$\left. + \left[\frac{1}{6}\left(\frac{a}{b}\right)^3 + \frac{1}{24}\left(\frac{a}{b}\right)^4 + \frac{1}{4}\left(\frac{a}{b}\right)^2 + \frac{1}{6}\left(\frac{a}{b}\right) + \frac{1}{24}\right](bt)^4 e^{-bt}\right\}$$

$$F_6(t) = \frac{-1}{b^2}\left\{\left(\frac{a}{b}\right)^4\left[\frac{-6a}{b} - 5 + at + [\frac{6a}{b} + 5 + 5(a+b)t]\, e^{-bt}\right]\right.$$
$$+ \left[\frac{5}{2}\left(\frac{a}{b}\right)^4 + 2\left(\frac{a}{b}\right)^5 - \frac{1}{2}\right](bt)^2 e^{-bt} + \left[\frac{5}{6}\left(\frac{a}{b}\right)^4 + \frac{1}{2}\left(\frac{a}{b}\right)^5 + \frac{5}{6}\left(\frac{a}{b}\right) + \frac{1}{2}\right](bt)^3 e^{-bt}$$
$$+ \left[\frac{5}{24}\left(\frac{a}{b}\right)^4 + \frac{1}{12}\left(\frac{a}{b}\right)^5 - \frac{5}{12}\left(\frac{a}{b}\right)^2 - \frac{5}{12}\left(\frac{a}{b}\right) + \frac{1}{8}\right](bt)^4 e^{-bt}$$
$$\left. + \left[\frac{5}{120}\left(\frac{a}{b}\right)^4 + \frac{1}{120}\left(\frac{a}{b}\right)^5 + \frac{1}{12}\left(\frac{a}{b}\right)^3 + \frac{1}{12}\left(\frac{a}{b}\right)^2 + \frac{1}{24}\left(\frac{a}{b}\right) + \frac{1}{120}\right](bt)^5 e^{-bt}\right\}$$

$$F_7(t) = \frac{1}{b^2}\left\{\left(\frac{a}{b}\right)^5\left[\frac{-7a}{b} - 6 + at + [\frac{7a}{b} + 6 + 6(a+b)t]\, e^{-bt}\right]\right.$$
$$+ \left[3\left(\frac{a}{b}\right)^5 + \frac{5}{2}\left(\frac{a}{b}\right)^6 + \frac{1}{2}\right](bt)^2 e^{-bt} + \left[\left(\frac{a}{b}\right)^5 + \frac{2}{3}\left(\frac{a}{b}\right)^6 - \left(\frac{a}{b}\right) - \frac{2}{3}\right](bt)^3 e^{-bt}$$
$$+ \left[\frac{1}{4}\left(\frac{a}{b}\right)^5 + \frac{1}{8}\left(\frac{a}{b}\right)^6 + \frac{5}{8}\left(\frac{a}{b}\right)^2 + \frac{3}{4}\left(\frac{a}{b}\right) + \frac{1}{4}\right](bt)^4 e^{-bt}$$

$$+\left[\frac{1}{20}\left(\frac{a}{b}\right)^5+\frac{1}{60}\left(\frac{a}{b}\right)^6-\frac{1}{6}\left(\frac{a}{b}\right)^3-\frac{1}{4}\left(\frac{a}{b}\right)^2-\frac{3}{20}\left(\frac{a}{b}\right)-\frac{1}{30}\right](bt)^5e^{-bt}$$

$$+\left[\frac{1}{120}\left(\frac{a}{b}\right)^5+\frac{1}{720}\left(\frac{a}{b}\right)^6+\frac{1}{48}\left(\frac{a}{b}\right)^4+\frac{1}{36}\left(\frac{a}{b}\right)^3+\frac{1}{48}\left(\frac{a}{b}\right)^2+\frac{1}{120}\left(\frac{a}{b}\right)+\frac{1}{720}\right](bt)^6e^{-bt}\Bigg\}$$

$$F_8(t)=\frac{-1}{b^2}\Bigg\{\left(\frac{a}{b}\right)^6\left[\frac{-8a}{b}-7+at+\left[\frac{8a}{b}+7+7(a+b)t\right]e^{-bt}\right]$$

$$+\left[\frac{7}{2}\left(\frac{a}{b}\right)^6+3\left(\frac{a}{b}\right)^7-\frac{1}{2}\right](bt)^2e^{-bt}+\left[\frac{7}{6}\left(\frac{a}{b}\right)^6+\frac{5}{6}\left(\frac{a}{b}\right)^7\right.$$

$$\left.+\frac{7}{6}\left(\frac{a}{b}\right)+\frac{5}{6}\right](bt)^3e^{-bt}+\left[\frac{7}{24}\left(\frac{a}{b}\right)^6+\frac{1}{6}\left(\frac{a}{b}\right)^7-\frac{7}{8}\left(\frac{a}{b}\right)^2\right.$$

$$\left.-\frac{7}{6}\left(\frac{a}{b}\right)-\frac{5}{12}\right](bt)^4e^{-bt}+\left[\frac{7}{120}\left(\frac{a}{b}\right)^6+\frac{1}{40}\left(\frac{a}{b}\right)^7\right.$$

$$\left.+\frac{7}{24}\left(\frac{a}{b}\right)^3+\frac{21}{40}\left(\frac{a}{b}\right)^2+\frac{7}{20}\left(\frac{a}{b}\right)+\frac{1}{12}\right](bt)^5e^{-bt}$$

$$+\left[\frac{7}{720}\left(\frac{a}{b}\right)^6+\frac{1}{360}\left(\frac{a}{b}\right)^7-\frac{7}{144}\left(\frac{a}{b}\right)^4-\frac{7}{72}\left(\frac{a}{b}\right)^3-\frac{7}{80}\left(\frac{a}{b}\right)^2\right.$$

$$\left.-\frac{7}{180}\left(\frac{a}{b}\right)-\frac{1}{144}\right](bt)^6e^{-bt}+\left[\frac{1}{720}\left(\frac{a}{b}\right)^6+\frac{1}{5040}\left(\frac{a}{b}\right)^7\right.$$

$$\left.+\frac{1}{240}\left(\frac{a}{b}\right)^5+\frac{1}{144}\left(\frac{a}{b}\right)^4+\frac{1}{144}\left(\frac{a}{b}\right)^3+\frac{1}{240}\left(\frac{a}{b}\right)^2+\frac{1}{720}\left(\frac{a}{b}\right)+\frac{1}{5040}\right](bt)^7e^{-bt}\Bigg\}$$

$$F_9(t)=\frac{1}{b^2}\Bigg\{\left(\frac{a}{b}\right)^7\left[\frac{-9a}{b}-8+at+\left[\frac{9a}{b}+8+8(a+b)t\right]e^{-bt}\right]$$

$$+\left[4\left(\frac{a}{b}\right)^7+\frac{7}{2}\left(\frac{a}{b}\right)^8+\frac{1}{2}\right](bt)^2e^{-bt}+\left[\frac{4}{3}\left(\frac{a}{b}\right)^7+\left(\frac{a}{b}\right)^8-\frac{4}{3}\left(\frac{a}{b}\right)-1\right](bt)^3e^{-bt}$$

$$+\left[\frac{1}{3}\left(\frac{a}{b}\right)^7+\frac{5}{24}\left(\frac{a}{b}\right)^8+\frac{7}{6}\left(\frac{a}{b}\right)^2+\frac{5}{3}\left(\frac{a}{b}\right)+\frac{5}{8}\right](bt)^4e^{-bt}$$

$$+\left[\frac{1}{15}\left(\frac{a}{b}\right)^7+\frac{1}{30}\left(\frac{a}{b}\right)^8-\frac{7}{15}\left(\frac{a}{b}\right)^3-\frac{14}{15}\left(\frac{a}{b}\right)^2-\frac{2}{3}\left(\frac{a}{b}\right)-\frac{1}{6}\right](bt)^5e^{-bt}$$

$$+\left[\frac{1}{90}\left(\frac{a}{b}\right)^7+\frac{1}{240}\left(\frac{a}{b}\right)^8+\frac{7}{72}\left(\frac{a}{b}\right)^4+\frac{7}{30}\left(\frac{a}{b}\right)^3+\frac{7}{30}\left(\frac{a}{b}\right)^2+\frac{1}{9}\left(\frac{a}{b}\right)\right.$$

$$\left.+\frac{1}{48}\right](bt)^6e^{-bt}+\left[\frac{1}{630}\left(\frac{a}{b}\right)^7+\frac{1}{2520}\left(\frac{a}{b}\right)^8-\frac{1}{90}\left(\frac{a}{b}\right)^5-\frac{1}{36}\left(\frac{a}{b}\right)^4-\frac{1}{30}\left(\frac{a}{b}\right)\right.$$

$$\left.-\frac{1}{45}\left(\frac{a}{b}\right)^2-\frac{1}{126}\left(\frac{a}{b}\right)-\frac{1}{840}\right](bt)^7e^{-bt}+\left[\frac{1}{5040}\left(\frac{a}{b}\right)^7\right.$$

$$+\frac{1}{40320}\left(\frac{a}{b}\right)^8+\frac{1}{1440}\left(\frac{a}{b}\right)^6+\frac{1}{720}\left(\frac{a}{b}\right)^5+\frac{1}{576}\left(\frac{a}{b}\right)^4$$

$$\left.+\frac{1}{720}\left(\frac{a}{b}\right)^3+\frac{1}{1440}\left(\frac{a}{b}\right)^2+\frac{1}{5040}\left(\frac{a}{b}\right)+\frac{1}{40320}\right](bt)^8e^{-bt}\Bigg\}$$

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